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COMPREHENSIVE EVALUATION OF MAJOR THEORIES ON THE ULTIMATE STRENGTH OF REINFORCED CONCRETE PANELS SUBJECTED TO IN-PLANE SHEAR FORCES AND THE PROPOSED SEMI-ANALYTICAL METHOD FOR ESTIMATION OF ULTIMATE STRENGTH

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Hiromichi YOSHIKAWA Hidetaka UMEHARA Tada-aki TANABE

SYNOPSIS

Major theories developed in the past on the in-plane shear load capacity of a regularly reinforced concrete panel are comprehensively compared. Some of the interrelationship between them are also demonstrated. Then the predictability of these theories are studied, comparing them with experimental results of 70 specimens in terms of dimensionless descriptions.

It is recognized that most of the values calculated by the various theories provide similar results and show good agreement with actual data when the ultimate failure is governed by the yielding of reinforcements. On the other hand close agreement is not obtained between observed and calculated values when the concrete crushes prior to the yielding of the reinforcements.

Finally a semi-analytical method to estimate the ultimate shear strength of an orthogonally reinforced concrete panel is proposed based on modified limit analysis methods for both cases.

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1. INTRODUCTION

Analytical approaches as well as experimental studies on the mechanical behavior of reinforced concrete panel-elements where in-plane stresses are dominant have been carried out intensively so far. Various theories have been developed each of them differing at the point which they put heavier stress on and at the method of deriving formulations.

It is first shown in this study that the in-plane shear strength of a reinforced concrete panel based on these various theories can easily and systematically be obtained through one common equilibrium condition and then transforming its coordinate system. It appears that differences between given formulae are stemming from the assumed stress state on concrete cracked surfaces.

It is noted that some models mentioned herein can form closed failure criteria for cracked and reinforced materials which may be analogous to yield function of isotropic materials in the elasto-plastic theory. The authors will finally propose a semi-analytical method to predict the ultimate shear strength which gives a better agreement with the experimental data collected based on the aforementioned studies.

2. COMPREHENSIVE EVALUATION OF PRINCIPAL THEORIES ON THE IN-PLANE SHEAR STRENGTH

The structural behavior of reinforced concrete panels is a reflected product of phenomena, such as a shear transfer along cracked surfaces, an inelastic resistance mechanism of reinforcement and concrete, and coupling effects of them. It is thus quite a complicated problem if one tries to predict the entire behavior of a reinforced concrete panel including deformational characteristics from an early elastic state up to the ultimate stage. On the other hand, when only the ultimate load capacity (maximum strength) of a reinforced concrete panel subjected to in-plane forces is concerned, analytical procedure becomes much simpler. Some theories such as limit analysis and other macroscopic modeling are considered to be the good examples of this.

Most of the major theories on the in-plane shear strength begin with different assumptions and derivations, and end up with seemingly different indexes and formulations. It is thus not so simple to compare them and distinguish the differences between them. It is required to transform or reexpress on the unified and common coordinate system in order to evaluate and to compare these theories comprehensively. Authors first try to indicate most of the formulae from various analytical models can be derived by the equilibrium condition alone and that difference between theories can be identified by a different assumed stress state.

2.1 Unified Formulation of Major Models Using Matrix Notation

Let us consider a reinforced concrete panel subjected to in-plane shear stresses as well as normal stresses, schematically shown in Fig. 1, which constitutes an earthquake-resisting component of many types of concrete structures. The reinforcing bars are orthogonally and regularly arranged in x and y direction. α is an angle between the principal stress direction and the x axis, and β is an inclination of cracks with respect to the y axis, which are taken to be positive in counterclockwise. It is customarily assumed that bond failure and local failure are negligible and that the spacing of reinforcing bars as well as the spacing of cracks is sufficiently dense so that a concrete element can be considered to be in a uniform stress state.

The stress state of reinforcing bars, the cracked concrete and the applied internal forces are represented by the following stress matrices in their own local coordinate systems, respectively.

$$[\bar{\sigma}_s] = \begin{bmatrix} R_x & 0 \\ 0 & R_y \end{bmatrix}, \quad [\bar{\sigma}_c] = \begin{bmatrix} \sigma_n & \tau_c \\ \tau_c & -\sigma_c \end{bmatrix}, \quad [\bar{F}_1] = \begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{bmatrix}, \quad [\bar{F}_2] = \begin{bmatrix} \sigma_x & \tau_{xy} \\ \tau_{xy} & \sigma_y \end{bmatrix} \tag{1}$$

Where $[\bar{\sigma}_s]$ is a stress matrix of regularly reinforcing nets in their longitudinal direction, i.e., in the x-y direction, $[\bar{\sigma}_c]$ is a stress matrix of cracked concrete along the cracked surface, and $[\bar{F}_1]$ and $[\bar{F}_2]$ are applied stress matrices in the principal direction of the forces and the x-y direction, respectively. Table 1 offers more detailed informations about these four stress matrices.

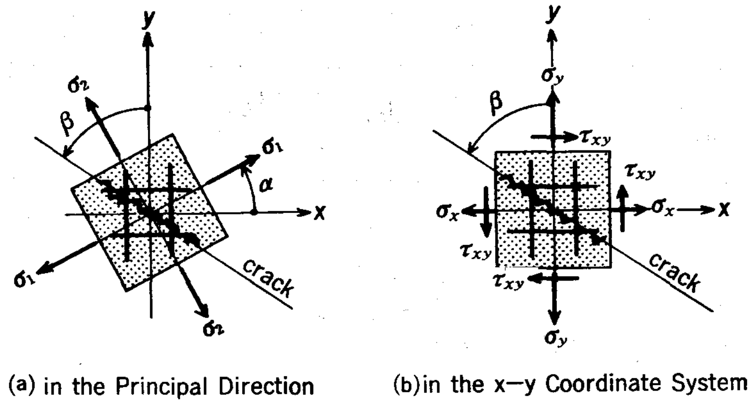


Fig. 1 Reinforced Concrete Panels subjected to In-Plane Forces

R_x and R_y are called "equivalent" stress of steel in x directional bars and y directional bars, respectively, defined by reinforcement ratios, P_x, P_y and the axial stress in reinforcing bars σ_{sx}, σ_{sy}, i.e., $R_x = P_x \sigma_{sx}$, $R_y = P_y \sigma_{sy}$. When equivalent stress of reinforcing bars in both directions are the same ($R = R_x = R_y$), R is referred to herein as isotropic reinforcing bars. In the case of cracked concrete σ_n is the normal stress perpendicular to the direction of the crack which is usually neglected, σ_c is the compressive stress parallel to the direction of the crack and τ_c is the shear stress along the crack. In general, directions of the above local coordinate systems are not identical and therefore the principal direction α or the cracking direction β is undetermined.

When deriving a formula for the ultimate strength, it is required to determine these stress matrices according to the failure type and assumptions. For example, $R = Pf_y$ when reinforcement yielding type or $\sigma_c = f_c$ when concrete fails. (f_c is the compressive strength of concrete and f_y is the yield strength of reinforcement). Coordinate transformation is employed to get a balanced condition (equilibrium condition) in a properly chosen common coordinate system as follows,

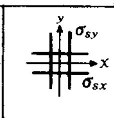
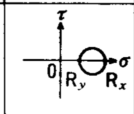
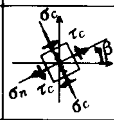
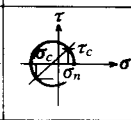
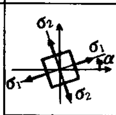
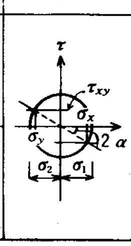
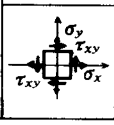
$$[\sigma_s]_\theta + [\sigma_c]_\theta = [F]_\theta \tag{2}$$

The transformation law for coordinate system is in general defined as,

$$[\sigma]_\theta = [T]^t[\bar{\sigma}][T] \;, \quad [T] = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \tag{3}$$

in which $[\bar{\ }]$ and $[\ ]_\theta$ express stress matrices in local coordinate and in transformed coordinate systems, respectively, and [T] is a transformation matrix. Final results are summarized in Table 1.

Table 1 Stress Matrices for Materials and Internal Forces

model		Mohr's circle	stress matrices: in local coordinate system	stress matrices: in transformed coordinate system
reinforcement			$[\bar{\sigma}_s]=\begin{bmatrix} R_x & 0 \\ 0 & R_y \end{bmatrix}$	$[\sigma_s]_\theta=\begin{bmatrix} R_x\cos^2\theta+R_y\sin^2\theta & (R_x-R_y)\sin\theta\cdot\cos\theta \\ (R_x-R_y)\sin\theta\cdot\cos\theta & R_x\sin^2\theta+R_y\cos^2\theta \end{bmatrix}$
cracked concrete			$[\bar{\sigma}_c]=\begin{bmatrix} \sigma_n & \tau_c \\ \tau_c & -\sigma_c \end{bmatrix}$	$[\sigma_c]_\theta=\begin{bmatrix} -\tau_c\sin 2\theta-\sigma_c\sin^2\theta & \tau_c\cos 2\theta+\frac{1}{2}\sigma_c\sin 2\theta \\ \tau_c\cos 2\theta+\frac{1}{2}\sigma_c\sin 2\theta & \tau_c\sin 2\theta-\sigma_c\cos^2\theta \end{bmatrix}$ ($\sigma_n=0$)
internal applied forces			$[\bar{F}_1]=\begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{bmatrix}$	$[F_1]_\theta=\begin{bmatrix} \sigma_1\cos^2\theta+\sigma_2\sin^2\theta & (\sigma_1-\sigma_2)\sin\theta\cos\theta \\ (\sigma_1-\sigma_2)\sin\theta\cos\theta & \sigma_1\sin^2\theta+\sigma_2\cos^2\theta \end{bmatrix}$
			$[\bar{F}_2]=\begin{bmatrix} \sigma_x & \tau_{xy} \\ \tau_{xy} & \sigma_y \end{bmatrix}$	$[F_2]_\theta=\begin{bmatrix} \sigma_x\cos^2\theta+\sigma_y\sin^2\theta-2\tau_{xy}\sin\theta\cos\theta & (\sigma_x-\sigma_y)\sin\theta\cos\theta+\tau_{xy}(\cos^2\theta-\sin^2\theta) \\ (\sigma_x-\sigma_y)\sin\theta\cos\theta+\tau_{xy}(\cos^2\theta-\sin^2\theta) & \sigma_x\sin^2\theta+\sigma_y\cos^2\theta+2\tau_{xy}\sin\theta\cos\theta \end{bmatrix}$

Most of the formulae proposed by the various researchers can be derived by means of the above equations (2) and (3) which may be different from their original methods, which will be developed below.

a. Nielsen .... Nielsen studied limit analysis to obtain analytical solutions for the ultimate strength of several types of concrete member ([1], [3], [7], [8]). As for a reinforced concrete panel in a membrane stress state, the following balanced equation is given choosing the x-y direction as a common coordinate system,

$$[\sigma_s]_{\theta=0} + [\sigma_c]_{\theta=-\beta} = [F_2]_{\theta=0} \tag{4}$$

Nielsen [8] assumes that the concrete is a uniaxial compressive member which carries a load only in the cracking direction. Therefore, $\sigma_n = 0$ and $\tau_c = 0$, and the above equation can be written as,

$$\begin{bmatrix} R_x & 0 \\ 0 & R_y \end{bmatrix} + \begin{bmatrix} -\sigma_c\sin^2\beta & -\frac{1}{2}\sigma_c\sin 2\beta \\ -\frac{1}{2}\sigma_c\sin 2\beta & -\sigma_c\cos^2\beta \end{bmatrix} = \begin{bmatrix} \sigma_x & \tau_{xy} \\ \tau_{xy} & \sigma_y \end{bmatrix} \tag{5}$$

When the reinforcement bars in both directions have reached a tensile yield point, the shear stress is given as follows from Eq. (5) considering that $R_x = p_x f_y$, $R_y = p_y f_y$ and $\sigma_c < f_c$.

$$\tau_{xy} = \sqrt{(p_x f_y - \sigma_x)(p_y f_y - \sigma_y)} \qquad (\sigma_c < f_c) \tag{6}$$

In the case of isotropic reinforcement ($R_x = R_y = R$) and a pure stress state, ($\sigma_x = \sigma_y = 0$), Eq. (6) becomes a simple equation, $\tau_{xy} = R = Pf_y$, which is identical with a formula used in ordinary design practice. When one of either of the reinforcements yield and the compressive failure of concrete occurs, the shear stress τ_{xy} is provided as follows,

$$\tau_{xy} = \sqrt{(p_y f_y - \sigma_y)\{f_c - (p_y f_y - \sigma_y)\}} \tag{7}$$

in which stress state is considered in such a way that $R_x < P_x f_y$, $R_y = P_y f_y$ and $\sigma_c = f_c$. Eq. (7) then reaches its maximum value,

$$\tau_{xy} = \frac{1}{2} f_c \tag{8}$$

when $R_y - \sigma_y = f_c/2$. Eq. (8) shows the maximum shear stress characterized only by the compressive strength of concrete f_c, which is in a so-called over-reinforced state. Finally, a continuation of the above development into the compressive yielding of reinforcement leads from the above formulae to the five formulae for the shear strength of a reinforced concrete panel which construct closed failure criteria to be mentioned later.

b. Marti .... Marti [13] introduced the modified Mohr-Coulomb criterion with tension-cutoff as a fracture behavior of concrete in limit analysis. Hence, by considering that the axial stress σ_n normal to the cracking direction is equal to the tensile strength of concrete f_t at failure and making the same development as Eqs. (4) through (8), Marti's shear strength for a reinforced concrete panel is obtained. For instance, the following equation is given,

$$\tau_{xy} = \sqrt{(p_x f_y + f_t - \sigma_x)(p_y f_y + f_t - \sigma_y)} \tag{9}$$

which corresponds to Eq. (6) by Nielsen.

c. Ono and Tanaka .... Ono and Tanaka's work [14] is known in Japan as an analytical approach using limit analysis and resulted in the upper and lower bound limitations for load carrying capacity of shear walls. They assumed that the cracked and reinforced concrete is isotropic and that the entire failure criteria can be determined from only the two values; the tensile strength T and the compressive strength C of a reinforced concrete member. (We assume that T = R and C = f_c + R, in which R is considered to be Pf_y, so that this analytical work can be compared with other theories.) So obtained failure criteria is subdivided into three regions, tensile, shear and compressive failure types, which will be shown in Fig. 2. The present study doesn't allow the use of Eq. (2) because the reinforced concrete is not regarded as a composite material in their theory. In case of an isotropic material, however, the transformation law, Eq. (3), makes it possible to obtain the shear strength as follows. The shear strength of tensile-failure type is written as,

$$\tau_{xy} = \sqrt{(pf_y - \sigma_x)(pf_y - \sigma_y)} \tag{10}$$

using $\sigma_1 = T$. In the case of shear failure type, one can get,

$$\tau_{xy} = \frac{\sqrt{\{(pf_y+f_c)(pf_y-\sigma_x) + pf_y\sigma_y\}\{(pf_y+f_c)(pf_y-\sigma_y) + pf_y\sigma_x\}}}{2pf_y + f_c} \tag{11}$$

using $\sigma_1/T - \sigma_2/C = 1$. Eq. (10) is identical with the case of isotropic reinforcement of Eq. (6), while Eq. (11) appears quite different from the corresponding equation given by Nielsen.

d. Bazant and Tsubaki .... Bazant and Tsubaki [15] proposed the slip-free (frictional) criterion for an orthogonally reinforced and arbitrarily cracked concrete element failing by the tensile yielding of both reinforcements basing on limit theory. Their study notes that any shear stress transmitted across the crack must be accompanied by the occurrence of significant normal compressive stress in the concrete perpendicular to the crack surface, and of the corresponding tensile stress in the reinforcement due to cracking dilatancy. Therefore, the tensile stress σ_n must be replaced by the compressive stress of concrete $-\sigma_c^*$, namely,

$$[\bar{\sigma}_c] = \begin{bmatrix} -\sigma_c^* & \tau_c \\ \tau_c & -\sigma_c \end{bmatrix} \qquad (12)$$

The condition of no-slip along the cracked surface is expressed in the form of Coulomb law (frictional criterion) such that,

$$\tau_c < k\sigma_c^* \quad \text{(k : frictional coefficient on cracked surfaces)} \qquad (13)$$

Eq. (2) can be written in the form as,

$$[\sigma_s]_{\theta=\beta} + [\sigma_c]_{\theta=0} = [F_1]_{\theta=\beta-\alpha} \qquad (14)$$

by selecting the cracking direction β as a common coordinate system, then rearranging Eqs. (12) to (14) leads to the following inequality,

$$g(\beta) = \{k(1+\cos2\beta)-\sin2\beta\}p_x f_y + \{k(1-\cos2\beta)+\sin2\beta\}p_y f_y - k(\sigma_1+\sigma_2) \\ -(\sigma_1-\sigma_2)\{k\cos2(\beta-\alpha)-\sin2(\beta-\alpha)\} > 0 \qquad (15)$$

To obtain safe design for all possible cracking directions, two conditions such as,

$$g(\beta) = 0 , \quad \frac{dg(\beta)}{d\beta} = 0 \qquad (16)$$

are imposed on Eq. (15). From Eqs. (15) and (16) the following shear criterion is finally obtained,

$$\tau_{xy} = \frac{1}{2\sqrt{r_2}} \{(p_x f_y-\sigma_x)-r_1(p_y f_y-\sigma_y)\}\{(p_y f_y-\sigma_y)-r_1(p_x f_y-\sigma_x)\} \qquad (17)$$

in which $r_1 = 1-\sin\gamma/1+\sin\gamma$ and $r_2 = 1/1+\sin\gamma$ ($\gamma = \arctan(k)$), being characterized by the friction coefficient k. It should be noted that the above equation reduces to Eq. (6) by Nielsen for $k\rightarrow\infty$.

e. Baumman .... The coordinate system in the direction of reinforcements (the x-y direction) is the proper choice of coordinate axes if one wants to obtain the equilibrium equation by Baumann [16]. Namely, it is,

$$[\sigma_s]_{\theta=0} + [\sigma_c]_{\theta=-\beta} = [F_1]_{\theta=-\alpha} \tag{18}$$

Assuming that $\sigma_n = 0$ and $\tau_c \neq 0$ in the concrete stress matrix, one has,

$$\begin{aligned} p_x f_y &= \sigma_1 \cos^2\alpha(1+\tan\alpha\tan\beta) + \sigma_2 \sin^2\alpha(1-\cot\alpha\tan\beta) - \tau_c \tan\beta \\ p_y f_y &= \sigma_1 \sin^2\alpha(1+\cot\alpha\cot\beta) + \sigma_2 \cos^2\alpha(1-\tan\alpha\cot\beta) + \tau_c \cot\beta \\ \sigma_c &= (\sigma_1 - \sigma_2)\frac{\sin 2\alpha}{\sin 2\beta} + 2\tau_c \cot 2\beta \end{aligned} \tag{19}$$

These equations are identical with ones provided by Baumann [16]. Since the principal stress direction of concrete β becomes the indeterminate value before the yielding of reinforcements, another equation is needed. For this requirement, Baumann introduced a strain energy approach [16] and Aoyagi [17] proposed the experimentally obtained formula by which β can be calculated from the ratio of principal stresses σ_2/σ_1 and the reinforcement direction α. On the other hand, the ultimate shear stress can be obtained from Eqs. (19) alone when the reinforcing bars in both directions reach the yield strength. For example, in the case of the isotropic reinforcement which results in $\beta = \alpha$, Eqs. (19) reduce to Eq. (10).

f. Collins .... In the diagonal compression field theory formulated by Collins [19], he utilizes the compatibility condition as well as the equilibrium condition so that the deformation can be analyzed. It is assumed in his theory that for the diagonally cracked concrete the direction of the principal stress coincides with that of the principal strain and a constitutive law relating the two values is simply linear. Although the diagonal compression field theory has originally been built for understanding the shear behavior of a prestressed or reinforced concrete beam, it is considered to be also applicable to the problem treated in this paper.

In stress field that Collins treated the shear stress plus the axial stress to one coordinate direction are considered. Thus the following equation may be written to fulfill these conditions,

$$\begin{bmatrix} p_x f_y & 0 \\ 0 & p_y f_y \end{bmatrix}_{\theta=0} + \begin{bmatrix} 0 & 0 \\ 0 & -\sigma_c \end{bmatrix}_{\theta=\beta} = \begin{bmatrix} \sigma_0 & \tau_{xy} \\ \tau_{xy} & 0 \end{bmatrix}_{\theta=0} \tag{20}$$

Then, when $R_x = p_x f_y$ and $R_y = p_y f_y$, you can have,

$$\tau_{xy} = \sqrt{(R_x - \sigma_0) R_y} \tag{21}$$

from Eq. (20). When the compressive failure of concrete precedes the yielding failure of reinforcing bars, the following equation is also obtained from Eq. (20) on the assumptions that $R_x < p_x f_y$, $R_y \leqq p_y f_y$ and $\sigma_c = f_{du}$,

$$\tau_{xy} = \sqrt{R_y(f_{du} - R_y)} \tag{22}$$

Eqs. (21) and (22) are in accordance with a special case of Eqs. (6) and (7) given by Nielsen, respectively. The limiting value of the compressive stress in the diagonal member, f_{du} in Eq. (22), is considered to be lower than the standard cylinder crushing strength f_c. Collins proposed the formula for this limiting value f_{du} expressed in terms of the maximum shear strain, which will be stated in 2.3.

g. Duchon .... The theory formulated by Duchon [18] is also to analyze stresses in materials and deformation of reinforced or prestressed concrete members in a membrane state. According to the authors' calculations, a numerical result from Duchon's theory is similar to these by other theories.

Among the aforementioned seven theories, three theories which form closed-failure criteria in the plane stress are shown in Fig. 2. The envelope curves in the figure describe ultimate strength of orthogonally and isotropically reinforced concrete panels.

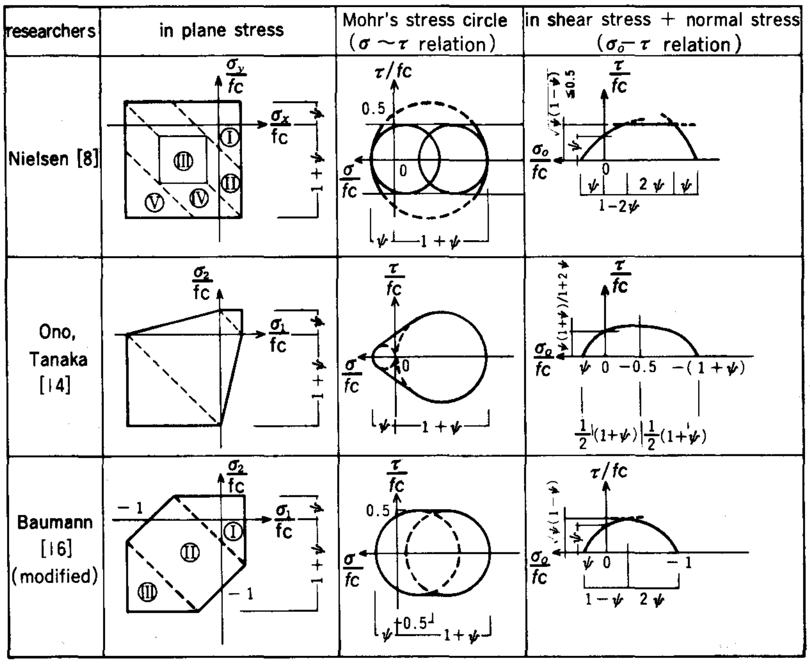


Fig. 2 Failure Criteria of a Reinforced Concrete Panel with Orthogonal and Isotropic Steel Bars

Note that in Fig. 2 dimensionless notations are employed, in which physical values are normalized relative to the compressive strength of concrete f_c, and that ψ, the degree of reinforcement, is defined as $\psi = Pf_y/f_c$ and that the diagrams are obtained taking the uniaxial tensile strength T to be equal to Pfy and the uniaxial compressive strength to be $C = f_c + Pf_y$ in Tanaka's theory and extending the failure criterion given by Baumann to the compressive failure region. In the case of tensile failure or compressive failure of reinforcement (which correspond to domain I and V in Nielsen's criteria) these three theories provide almost the same result, while the theories reveal different strengths in-between where the concrete fails prior to the steel yielding.

It may be seen from the investigation in this chapter that each of the major formulae for the ultimate strength of a reinforced concrete panel can be clearly derived solely from the equations with the Mohr's law and that the differences among the theories mainly depend upon the assumed stress matrix for the concrete. The interrelationship between the theories investigated herein are briefly summarized as follows.

Compared with a theory by Nielsen [7] who introduced limit analysis, Bazant and Tsubaki [15] add to it the slip-free criterion on cracked surface and Marti [13] uses Mohr-Coulomb law as a failure criteria of the concrete. The results by Ono and Tanaka [14] based upon limit analysis both in the tensile failure and in the compressive failure type exactly agree with Nielsen's theory, whereas the result in shear failure type between the first two types differs from other theories. In the diagonal compression field theory formulated by Collins [9] and formulae given by Duchon [18], the deformations as well as the strength can be analyzed, introducing the compatibility condition for strains and the simplified stress-strain relation for the concrete.

This analytical treatment for reinforced concrete members will be quite useful as it is expected that these explicitly expressed failure criteria for a composite material (a cracked reinforced concrete) can readily be incorporated into the elasto-plastic theory as a yield surface.

2.2 Numerical comparisons of the theories

Numerical calculations are carried out for the previously mentioned theories whose assumptions and derivations were investigated in 2.1. To allow a general comparison of the ultimate in-plane shear strength, the non-dimensional values normalized by dividing by the compressive strength of concrete f_c are used, such that,

$$\eta = \frac{\tau}{f_c}, \quad \xi = \frac{\sigma}{f_c}, \quad \psi = \frac{pf_y}{f_c} \tag{23}$$

by which all the mathematical expressions obtainable from various models can be represented. Eq. (6) given by Nielsen, for example, is rewritten as,

$$\eta = \sqrt{(\psi_x - \xi_x)(\psi_y - \xi_y)} \tag{6'}$$

By adoption of these dimensionless descriptions, it is anticipated that formulae expressed by different units and measured data under different conditions can successfully be compared in a common base. (Hereafter τ indicates the shear stress at the ultimate stage, so subscripts xy are omitted, and R indicates the equivalent stress of reinforcing steel when yielded, $R = Pf_y$)

Comparisons of the ultimate strength calculated from the five theories are made in Fig. 3 for the state of pure shear stress and in Fig. 4 for the state of the shear stress plus the additional normal stress in the X direction.

Fig. 3 shows that compared with the values of Nielsen's limit analysis that precisely agrees with the conventionally used formula ($\tau = Pf_y$), the shear strength of Bazant's slip-free criteria gives smaller values with decrease of friction coefficient k and Marti's result varies according to the level of tensile strength of concrete. (The result given by Bazant at $k = \infty$ or the results by Marti at $\zeta = 0$ reduces to Nielsen's as stated early.) Limit theory formulated by Ono and Tanaka provides a different result from other theories partly because Mohr-Coulomb criterion is introduced as shear failure condition in their theory.

Fig. 4 depicts the interaction curves between the normalized in-plane shear strength and the normal stress when the degree of reinforcement is fixed at 0.2. All the curves start at $\xi_0 = \psi$ ($\sigma_0 = pf_y$) in the tensile region ($\xi_0 > 0$) and the shear strength increases with an increase of the compressive normal stress

($\xi_0 < 0$), levelling off or falling within the compressive failure domain when an over-reinforced state is reached. It is shown in Fig. 4 that all the theories provide similar results in the tensile failure region in which the yield of reinforcement precedes the concrete crush, while they differ significantly under the larger compressive normal stress where the concrete fails prior to the reinforcement yielding.

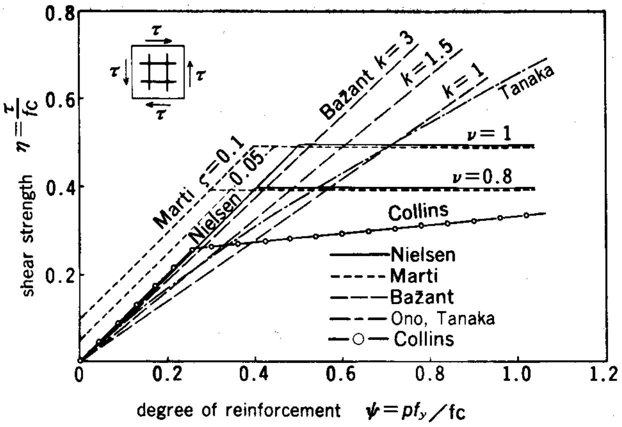


Fig. 3 Comparison of Calculated Values, ψ - η Relation in Pure Shear Stress

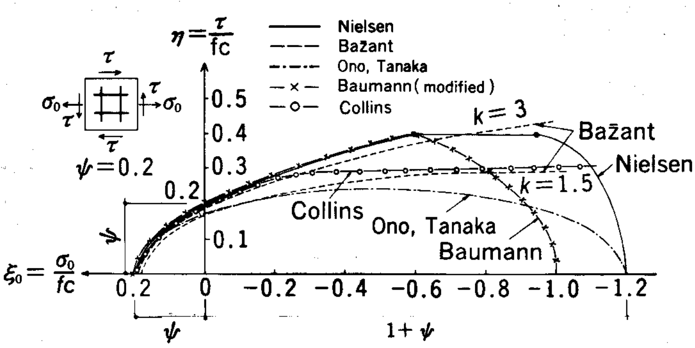


Fig. 4 Comparison of Calculated Values, ξ_0- η Relation in Shear Stress and One Directional Normal stress

2.3 The Shear Strength determined by the Compressive Failure of Concrete

In the over-reinforced state of a concrete member, where the concrete crush precedes and the increase of strength is not expected by the increase of the ratio of reinforcement, it is rather difficult to apply analytical approach and much scattering of experimental data is observed. In this situation, it seems to be decisively important how σ_c is assumed in the concrete stress matrix $[\bar{\sigma}_c]$ among Eq. (1). Limit analyses given by Nielsen and Marti offer Eq. (8) as an ultimate shear strength on the natural assumption that $\sigma_c = f_c$, which is identical to that by the extended Baumann's theory (see the last row in Fig. 2). While some researchers assume that the concrete in a structure crushes below its uniaxial strength as determined from standard cylinder specimens introducing the effectiveness factor of concrete strength.

Therefore, we use the following equations instead of Eq. (8).

$$\eta = \frac{1}{2}\nu \quad \text{or} \quad \tau = \frac{1}{2}\nu fc \quad , \quad (0 < \nu < 1) \tag{24}$$

The introduction of the effectiveness factor of the concrete ν may allow a general discussion of the shear strength of a reinforced panel due to the concrete failure.

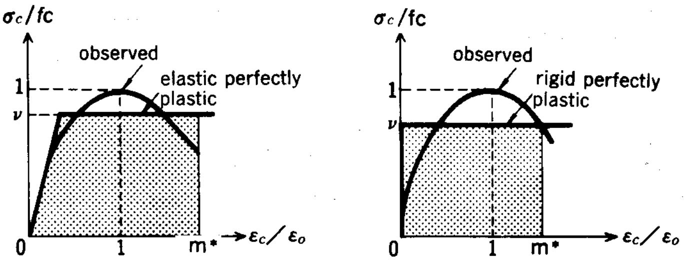
The effectiveness factor of the concrete can be figured out directly from shear tests using over-reinforced concrete members. The values $\nu = 0.80$ (Campbell [6]) and $\nu = 0.74$ (Braestrup [1]) are experimental values so obtained. Assuming it to be particularly related to the compressive strength of concrete f_c, Nielsen [3] and Higai [4] proposed the following equations,

$$\text{Nielsen : } \nu = 0.8 - f_c/2040 \qquad \text{Higai : } \nu = 10/\sqrt{f_c} \sim 13/\sqrt{f_c} \tag{25}$$

Exner [5] and Yoshikawa et al [12] considered that the concrete strength is required to be reduced in order that the strain energy doesn't change even if the concrete is assumed to be perfectly plastic in limit analysis (Fig. 5). Exner, then, arrived at,

$$\nu = 10.22/\sqrt{f_c} \tag{26}$$

which is again a single-valued function of the compressive strength of concrete, while Yoshikawa et al offer a constant value of $\nu = 0.75$ from his assumed parabolic stress-strain curve.



(a) Exner [5] (b) Yoshikawa [12]

Fig. 5 Perfect Plastic Model of Concrete

There are emprically determined values of the effectiveness factor from some tests on concrete containment vessels. From a series of verification tests on reinforced or prestressed concrete vessels conducted in Japan, Japanese design criteria for a concrete containment has been drafted (Ohsaki, et al [10]), providing $\tau = 5.25\sqrt{f_c}$ as a ceiling value of the ultimate shear strength. Analogous to Eq. (24), this value can be written as follows,

$$\tau = \frac{1}{2}\nu^* f_c \ , \quad \nu^* = 10.5/\sqrt{f_c} \tag{27}$$

It is also possible to connect other experimental formulae $\tau = 5.6\sqrt{f_c}$ (Ogaki et al [11]) and $\tau = 4.2\sqrt{f_c}$ (Kawamata and Iida et al [21]) with Eq. (24) as well. According to the investigation by Collins and Vecchio [19], the concrete strength of the diagonal compression member f_{du} in a shear member can be expressed in terms of maximum co-existing shear strain γ_m. A degrading factor

f_{du}/f_c proposed by them also corresponds to the effectiveness factor discussed herein, which is also described as,

$$\nu^{**} = \frac{f_{du}}{f_c} = \frac{3.6}{1+2\gamma_m/\varepsilon_0} \tag{28}$$

The above value may change depending upon a reinforcement ratio and stress state of a member, by which Collins' equation (28) is distinguished from other formulae all of which are characterized by the compressive strength of concrete alone.

3. COMPARISONS WITH TEST RESULTS

Values predicted by the theories stated so far are compared to the experimental data, which are presently available from approximately seventy specimens by eight research groups that are listed in Table 2.

Table 2 Major Experimental Works on In-Plane Shear Behavior

No.	Experiments	Variables	Researchers	References
A	Pure shear loading on RC panels	P_x, P_y, f_y	Vecchio, Collins	[19]
B	Normal forces loading on RC panels	α, ratio of σ_2/σ_1 P_x, P_y	Aoyagi, Yamada	[20]
C	Torsional loading with vertical axial force on RC cylinders	P, σ_y (axial force)	Yoshikawa, Iida, et al	[12]
D	Torsional loading with internal pressure on RC cylinders	P_x, P_y, σ_x (pressure level)	Nakayama	[22]
E	Torsional loading with internal pressure on PC cylinders	P_x, P_y σ_x, σ_y (prestress)	Kobayashi, Ogaki, et al	[23]
F	Torsional loading on RC cylinders	P_x, P_y, arrangement of steel	Uchida, Aoyagi, et al	[24]
G	Torsional and axial loading on RC cylinders	P, σ_y (axial stress)	Tsuboi, Tomii	[25]
H	Shear loading on RC panels with biaxial tensioned reinforcements	Initial biaxial tension	Oesterle, Russell	[26]

These tests are either torsional loading on a hollow cylindrical concrete specimen or in-plan forces loading on a flat concrete panel where reinforcement ratios and membrane stresses induced by mechanically applied forces, internal pressure or prestressing forces are chosen as changing parameters. Results obtained from horizontal loading tests using shear walls or hollow cylinder specimens are excluded here because stresses observed in those experimental works are not uniformly distributed over a specimen, which makes it difficult to convert force into stress directly. All of the studied specimens have orthogonally and regularly arranged reinforcing nets in either double or single layer. Failure of reinforced concrete panels is supposed to be divided into three modes as follows: both of reinforcing bars in two directions yield (mode

I); reinforcing bars in one direction yield and concrete crushed (mode II); and concrete crushed with no reinforcement bars yielded (mode III).

The ultimate shear strength in a pure shear stress state ($\psi - \eta$ relation) and the ultimate shear strength under shear stress with normal stress in one direction ($\xi_0 - \eta$ relation) are shown in Fig. 6 and Fig. 7, respectively, in which a dimensionless notation is again used. In order to carry out numerical calculations of each theory, it is necessary to determine some coefficients and constants required for the theories. It is, therefore, assumed that the effectiveness factor of concrete in Nielsen's theory is $\nu = 0.67$, the ratio of tensile and compressive strength of concrete in Marti's theory is $\zeta = 0.05$, the frictional coefficient on the cracked surfaces used in slip-free criterion is $k = 1.7$ and in limit analysis by Tanaka the tensile strength T and the compressive strength C as a reinforced concrete member are taken as $T = \psi f_c$ and $C = (1 + \psi) f_c$, respectively.

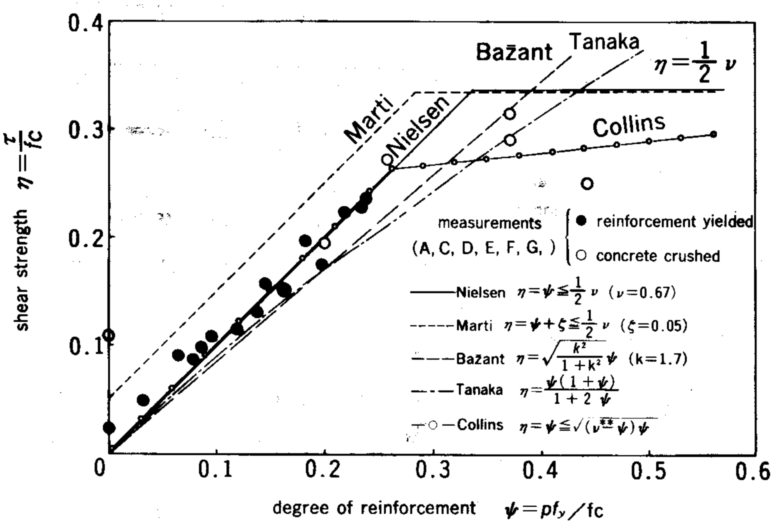
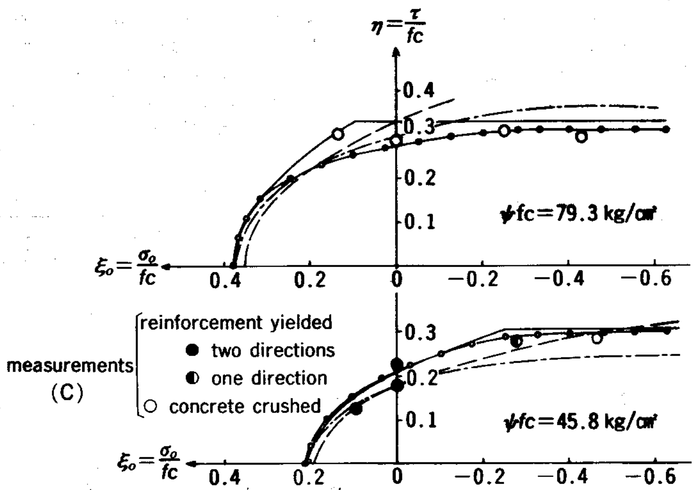


Fig. 6 Comparison with Test Results, $\psi - \eta$ Relation in Pure Shear Stress

Fig. 6 shows that measurements roughly agree with calculated values by Nielsen and a conventional formula $\tau = Pf_y$ and it may be said that a measurement slope in mode I is much closer to that of Bazant's slip-free criterion and an intercept of measured data with the vertical axis is not zero, which corresponds to ζ used in Marti's theory. These may be suggestive that the adoption of Mohr-Coulomb failure criterion is justified in the lower degree of reinforcement and the tensile stress in the reinforcement bars is increased due to slippage along cracked surfaces.

The regression curve, $\eta = 0.85\psi + 0.024$, is obtained from the experimental data in mode I of Fig. 6, which means the frictional coefficient $k = 1.6$ in Bazant's theory and the ratio of tensile-compressive strength of concrete $\zeta = 0.024$ in Marti's theory. Fig. 6 also shows that limit analysis by Tanaka provides the lower shear strength than the measured values through failure mode I and mode II.

Fig. 7 describes a change of the shear strength with respect to the normal stress for the case with a constant reinforcement ratio. (It should be noted however that the degrees of reinforcement ratios $\psi = Pf_y/f_c$ of the plotted measurements are not exactly the same from specimen to specimen because the compressive strengths of concrete f are slightly different.) As can be seen in Fig. 7, values calculated from the theories agree fairly well with experimental data, and all the theoretical predictions give a reasonable explanation of a change of in-plane shear strength through failure modes I, II and III.



$$\text{Nielsen} \quad \eta = \begin{cases} \sqrt{\psi(\psi-\xi_0)} \\ \sqrt{\psi(\nu-\psi)} \leq \frac{1}{2}\nu \end{cases}$$

$$\text{Bažant} \quad \eta = \frac{1}{2/r_2} \sqrt{\{(1-r_1)\psi-\xi_0\}\{(1-r_1)\psi+r_1\xi_0\}}$$

$$\text{Ono, Tanaka} \quad \eta = \sqrt{\psi(1+\psi)(\psi-\xi_0)(1+\psi+\xi_0)} / 1+2\psi$$

$$\text{Collins} \quad \eta = \begin{cases} \sqrt{\psi(\psi-\xi_0)} \\ \sqrt{\psi(\nu^{**}-\psi)} \end{cases}$$

Fig. 7 **Comparison with Test Results (ξ_0 - η Relation in** Shear Stress and One Directional Normal Stress)

The experimental result of mode III (concrete-crushed failure) are plotted with respect to the degree of reinforcement ψ and the compressive strength of concrete f_c in Figs. 8(a) and 8(b), respectively, in which values computed by the theories and by some proposed experimental equations are shown on the basis of the statement made in the section 2.3.

It can be seen in Figs. 8(a) and (b) that only Collin's prediction is influenced by the axial stress level and the degree of reinforcement and other theories and formulae depend upon the compressive strength of concrete only and that the theoretical predictions overestimate the experimental results. It is suggested that no theory is in close agreement with the experimental data and that the in-plane shear strength is slightly affected by the degree of reinforcement and the compressive strength of concrete.

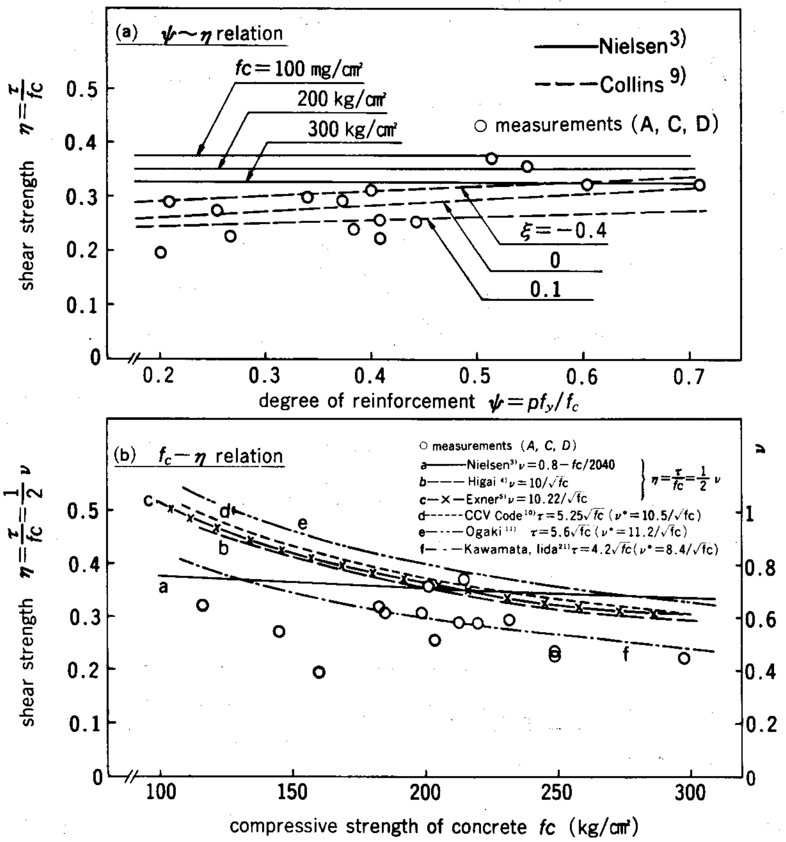


Fig. 8 In-Plane Shear Strength for Concrete-Crushed Failure; (a) $\psi - \eta$ Relation, (b) $f_c - \eta$ Relation

4. A SEMI-ANALYTICAL METHOD TO ESTIMATE THE IN-PLANE SHEAR STRENGTH

To improve the differences between the theoretical and the experimental results authors propose the following semi-analytical procedure for the estimation of in-plane shear strength of a reinforced concrete panel. This method is obtained by simplifying limit analyses formulated by Nielsen [7], Marti [13] and Bazant [15] (hereafter, equations in parenthese are expressed by a metric unit not a dimensionless expression) and making a correlation with experimental observation studied in the paper. Proposed formulae are prepared for reinforcement-yielded type (failure modes I and II) as well as concrete-crushed type (failure mode III).

Reinforcement-yielded type

If one takes,

$$\eta = a\psi + b\,, \quad (\tau = aR + bf_c) \tag{29}$$

as a general form of a shear strength formula for a reinforced concrete panel in a pure shear stress state, then this formula can represent all the three analytically obtained expressions by Nielsen, Marti and Bazant just by changing constants a and b, that is schematically shown in Fig. 9. The above equation corresponds to Eq. (16) by Nielsen when a = 1 and b = 0, and Eq. (29) is identical with Marti's theory (Eq. (9)) when a = 1 b = ζ, and with Bazant's theory (Eq. (11)) when a = $k/\sqrt{1+k^2}$ and b = 0.

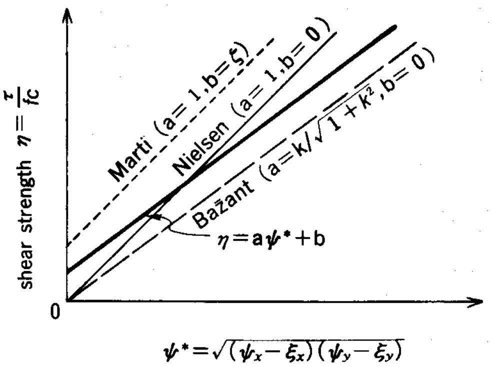


Fig. 9 Proposed Linear Equation for Shear Strength and Corresponding Theories (Reinforced-Yielded Type)

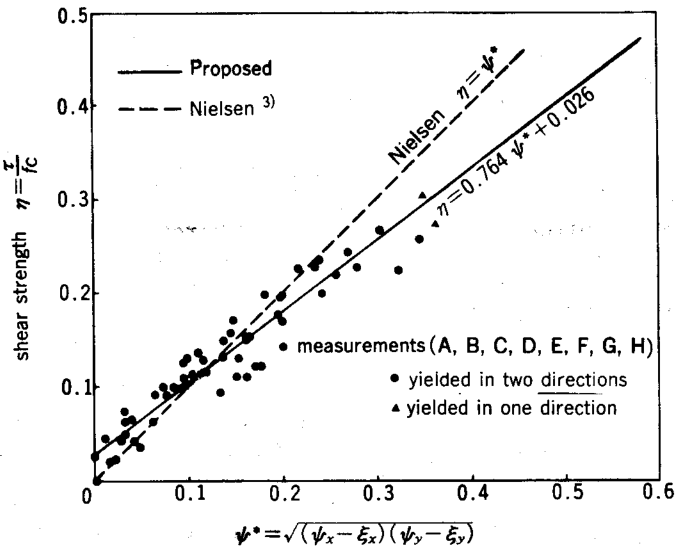


Fig. 10 Relationship between the Equivalent Degree of Reinforcement ψ^* and Shear Strength η (Reinforcement-Yielded Type)

We now introduce a new term, the "equivalent" degree of reinforcement, ψ^*, in the more extended form as,

$$\psi^* = \sqrt{(\psi_x-\xi_x)(\psi_y-\xi_y)} \quad , \qquad (R^* = \sqrt{(p_x f_y-\sigma_x)(p_y f_y-\sigma_y)}) \qquad (30)$$

This is determined from four quantities such as the reinforcement ratios and the applied normal stresses in both x and y directions. It is considered that the coefficients may be determined by adjusting them to the experimental data, supposing Eq. (29) is still applicable even when ψ is substituted with this equivalent degree of reinforcement ψ^*. Toward that purpose, measurements plotted in Fig. 6 is added by other data from specimens subjected to normal stresses in either one or two directions and is shown in Fig. 10 with respect to the equivalent degree of reinforcement ψ^*. By the least squares method a linear regression line is then obtained whose parameters a = 0.76 and b = 0.026, which may be in accordance with the investigation for Fig. 6. Now a formula of the shear strength for the reinforcement-yielding type is written as,

$$\eta = 0.76\,\psi^* + 0.026, \quad (\tau = 0.76\,R^* + 0.026\,f_c) \tag{31}$$

Fig. 10 exhibits that so obtained equation (31) leads to the better agreement with experimental results arranged by the term of the equivalent degree of reinforcement ψ^* rather than the chosen three theories discussed in this paper.

Concrete-crushed type

Previous studies in the section 3 imply that none of the theories or experimental formulae seen in Fig. 8 can give a satisfactory explanation. It is observed from the experimental results that the shear strength governed by the concrete crushing is slightly influenced by the compressive strength of concrete f_c and the degree of reinforcement ψ. Hence, we introduce a new dimensionless parameter, $\psi^*(250/f\ )^{0.5}$, which is linearly related to the shear strength of this failure mode.

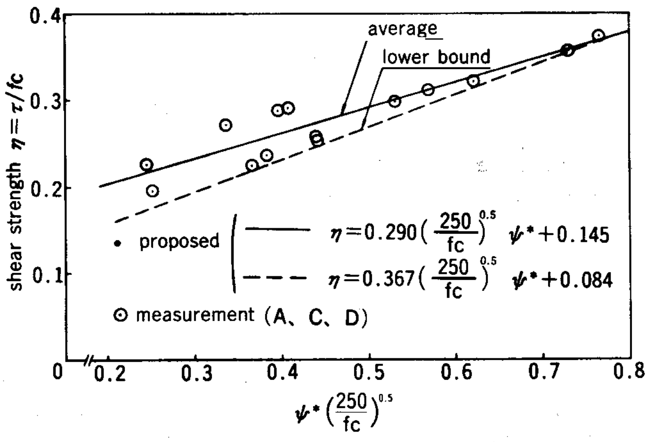


Fig. 11 Relationship between the Parameter $\psi^*(250/f_c)^{0.5}$ and Shear Strength η (Concrete-Crushed Type)

Consequently two straight lines are obtained as shown in Fig. 11; an estimated regression curve is indicated by a solid line and a lower bound is shown as a dashed line. The average strength (solid line) is thus expressed as,

$$\eta = 4.59\,\frac{\psi^*}{\sqrt{f_c}} + 0.145, \quad \left(\tau = 4.59\,\frac{R^*}{\sqrt{f_c}} + 0.145\,f_c\right) \tag{32}$$

5. THE USAGE OF THE NEWLY PROPOSED METHOD OF DETERMINING SHEAR STRENGTH

As for the application for the design use, one only needs to calculate the equivalent degree of reinforcement ψ^* and the parameter $\psi^*/f_c^{0.5}$ from given conditions and then two values of η (or τ) using Eq. (31) and Eq. (32). The smaller value of η (or τ) is the shear strength of the plane and the corresponding equation shows the failure mode, which is illustrated in Fig. 12 as a practical chart.

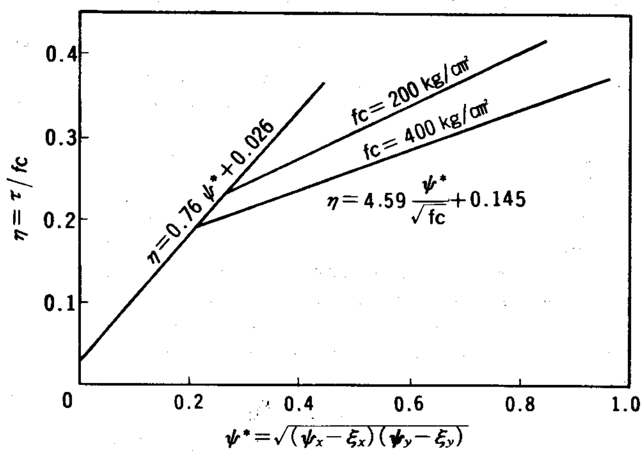


Fig. 12 Proposed Failure Criteria for In-Plane Shear Strength

6. CONCLUSION

The mechanical behavior of a reinforced concrete member failed in shear load are known to be influenced by a large number of factors, requiring a more sophisticated analytical method. Recent works therefore have a trend directed toward the more microscopic approaches, such as modeling the mechanism of the shear transfer and the the dowel action at the cracking using a finite element method.

However, it may be more useful if we can construct a practical yet accurate failure criteria with a simpler formulation. From that point of view, authors tried to evaluate major theories on the shear behaviors of reinforced concrete panels comprehensively and proposed failure criteria which is expressed with simple indexes in the form of the two concise equations; one corresponding to the reinforcement yielding mode and the other corresponding to the concrete crushing mode without losing accuracy as found in other theories.

Finally authors gratefully acknowledge the contributions of many researchers whose test results from their literature are referred to in this paper.

NOTATION

The principal symbols are defined as follows and others were indicated when necessary. Stress matrices are shown in detail in Table 1.

x, y	:	Directions of cartesian coordinate axes oriented in the longtitudinal directions of orthogonal reinforcing bars, subscript x and y refer to corresponding directions.
σ_1, σ_2	:	Principal stresses of internal applied forces.
$\{\sigma_x, \sigma_y, \tau_{xy}\}^t$	:	Stress components in the x-y direction.
f_y	:	Yield strength of reinforcement.
f_t	:	Tensile strength of concrete.
f_c	:	Compressive strength of concrete as determined from standard test cylinders.
ε_o	:	Compressive strain of concrete corresponding to f_c.
$[\bar{\sigma}_s]$, $[\sigma_s]_\theta$	:	Stress matrix of reinforcement.
$[\bar{\sigma}_c]$, $[\sigma_c]_\theta$	:	Stress matrix of cracked concrete.
$[\bar{F}_1]$, $[F_1]_\theta$	:	Stress matrix of internal applied stress in the principal direction.
$[\bar{F}_2]$, $[F_2]_\theta$	:	Stress matrix of internal applied stress in the x-y direction.
$[\bar{\ }]$	:	Stress matrix in local coordinate system.
$[\ ]_\theta$	:	Stress matrix in coordinate system transformed by θ, which is calculated from Eq. (3).
$[T]$	:	Transformation matrix.
p	:	Percentage of reinforcement.
R	:	Equivalent stress of reinforcement ($R = pf_y$ when yielded).
α	:	Angle between the principal direction and the x axis.
β	:	Inclination of a crack with respect to the y axis.
ψ	:	Degree of reinforcement defined as $\psi = pf_y/f_c$.
ξ	:	Normal stress normalized to the compressive strength of concrete, $\xi = \sigma/f_c$.
ξ_0	:	Constant and one directional normal stress normalized to the compressive strength of concrete, $\xi_0 = \sigma_0/f_c$.
η	:	Shear strength normalized to the compressive strength of concrete, $\eta = \tau/f_c$.
ζ	:	Ratio of tensile and compressive strength of concrete, $\zeta = f_t/f_c$.
k	:	Friction coefficient on a cracked surface.
r_1, r_2	:	Constants determined from k
ν	:	Effectiveness factor of concrete ($0 < \nu < 1$).
ψ^*	:	Equivalent degree of reinforcement, defined as $\psi^* = \sqrt{(\psi_x - \xi_x)\ (\psi_y - \xi_y)}$
R^*	:	Extended R, defined as $R^* = \sqrt{(p_x f_y - \sigma_x)\ (p_y f_y - \sigma_y)}$

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